Carleton UNIVERSITY
Canada's Capital University

Mathematics Tutorial Series
Integral Calculus #14

Integration and Differential Equations

Suppose we have a model:

$$\frac{dy}{dx} = 2xy$$

How can we solve for y?
How many different solutions exist?

Write the model as:

$$\frac{1}{y}\frac{dy}{dx} = 2x$$

The anti-derivatives of the left should equal the anti-derivatives of the right side.

So:

$$\int \frac{1}{y}\frac{dy}{dx}\,dx = \int 2x\,dx$$

The right side is easy:

$$\int 2x\,dx = x^2 + C$$

The left side looks like the result of a chain rule derivative.

$$\int \frac{1}{y}\frac{dy}{dx}\,dx = \log y + C$$

So:

$$\log y + C = x^2 + C$$

We only need one constant:

$$\log y = x^2 + C$$

So the solutions of the DE

$$\frac{dy}{dx} = 2xy$$

are

$$\log y = x^2 + C$$

$$y = e^{x^2+C} = e^c e^{x^2}$$

If the population was $y = e$ when $x = 0$, then $C = 1$ and the final solution is:

$$y = e^{x^2+1}$$

This is a **separable** differential equation.

$$\frac{1}{y}\frac{dy}{dx} = 2x$$

$$\frac{1}{y}\ dy = 2x\ dx$$

If you can separate a differential equation then you can use integrals to solve it.

Most differential equations are **non-separable**!!